Table of Situation Prompts

Content Categories:
Algebra: A Geometry: G Statistics: S
Number: N Trigonometry: T Calculus: C

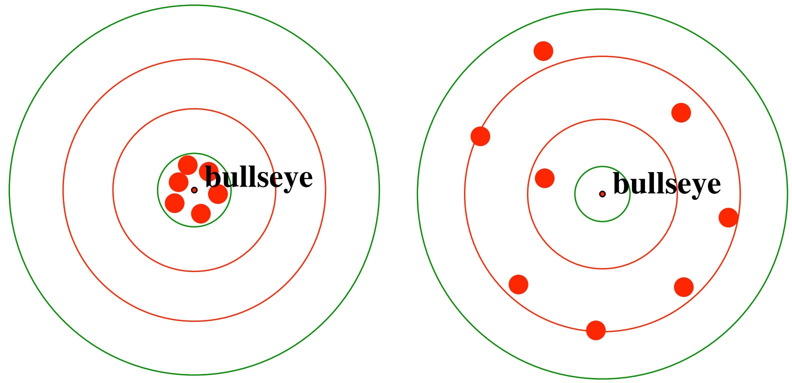
Status:
F (Final), I (intermediate), P (Prompt only)

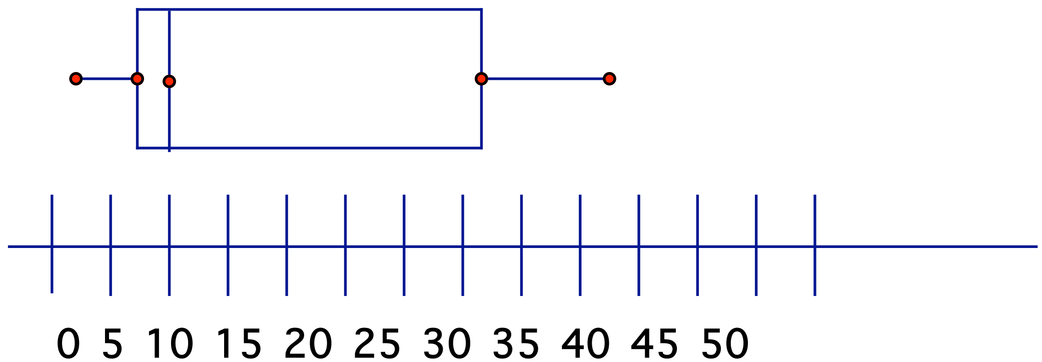
#	Title/Date		Prompt
01	Calculation of Sine 32	T	After completing a discussion on special right triangles (30°-60°-90° and 45°-45°-90°), the teacher showed students how to calculate the sine of various angles using the calculator. A student then asked, "How could I calculate sin (32°) if I do not have a calculator?"
02	Parametric Drawings	A	This example, appearing in CAS-Intensive Mathematics (Heid and Zbiek, 2004)***1,*** was inspired by a student mistakenly grabbing points representing both parameters (A and B in f(x)= Ax + B) and dragging them simultaneously (the difference in value between A and B stays constant). This generated a family of functions that coincided in one point. Interestingly, no matter how far apart A and B were initially, if grabbed and moved together, they always coincided on the line x = -1.

<table>
<tr><td>03</td><td>Inverse Trigonometric Functions</td><td>T</td><td>Three prospective teachers planned a unit of trigonometry as part of their work in a methods course on the teaching and learning of secondary mathematics. They developed a plan in which high school students would first encounter what the prospective teachers called “the three basic trig. functions”: sine, cosine, and tangent. The prospective teachers indicated in their plan that students next would work with “the inverse functions,” identified as secant, cosecant, and cotangent.</td></tr>
<tr><td>04</td><td>Representing Standard Deviation</td><td>S</td><td>In prior lessons, students learned to compute mean, mode and median. The teacher presented the formula for standard deviation and had students work through an example of computing standard deviation with data from a summer job context. The following written work developed during the example:

$$\begin{array}{cccc} x & \bar{x} & (x-\bar{x}) & (x-\bar{x})^2 \\ 140 & 200 & -60 & 3600 \\ 190 & 200 & -10 & 100 \\ 210 & 200 & 10 & 100 \\ 260 & 200 & 60 & 3600 \\ & & \sum(x_i-\bar{x})^2 = & 7400 \end{array}$$
$$\sigma = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n}}$$
$$\sigma = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}} \qquad \sigma = \sqrt{1850}$$
$$\sigma = \sqrt{\frac{7{,}400}{4}} \qquad \sigma \approx 43.01$$

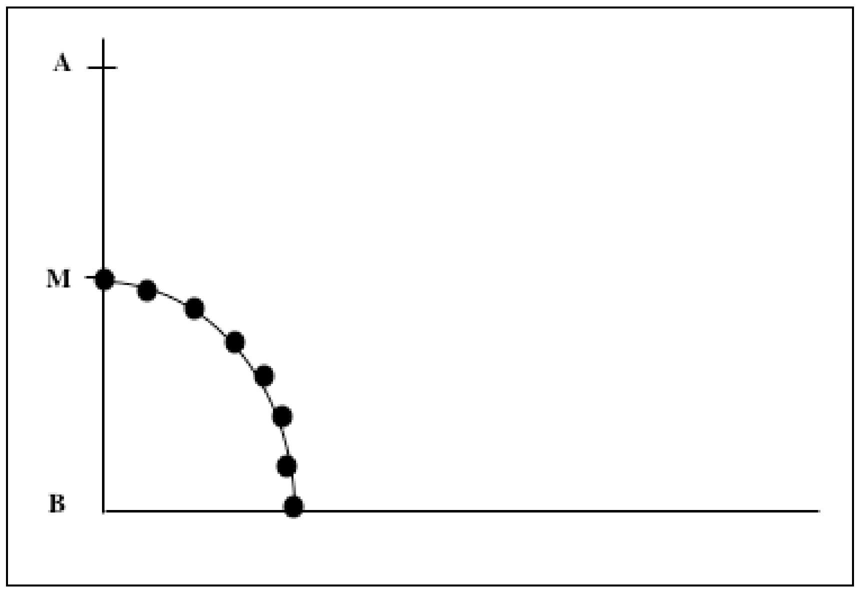
The teacher then said, “Standard deviation is a measure of the consistency of our data set. Do you know what consistency means?” To explain “consistency” the teacher used the idea of throwing darts. One student pursued the analogy, “If you hit the bull’s eye your standard deviation would be lower. But if you’re all over the board, your standard deviation would be higher.” The student drew the following picture to illustrate his idea:</td></tr>
</table>

			 A student raised her hand and asked, “But what does this tell us about what we are trying to find?”

05	What Good are Box and Whisker Plots?	S	The teacher had just reviewed mean, mode, and median with average seventh grade class before introducing the students to box-and-whisker plots. The teacher had written the following data set representing a football team's scores for each game of the season on the board: 3, 4, 7, 7, 7, 10, 13, 22, 32, 37, 44 The teacher and students collaboratively computed the mean, mode, median, and quartiles for the data. They used this information as the teacher walked the students through a handout that explained how to construct a box-and-whisker plot:  A student asks, "What does the box-and-whisker-plot stand for?" Another person adds, "What can you use it for?"

06	Can You Always Cross Multiply?	A	This is one of several lessons in an algebra I unit on simplifying radical expressions. The teacher led students through several examples of how to simplify radical expressions when the radicands are expressed as fractions. The class is in the middle of an example, for which the teacher has written the following on the whiteboard: $\sqrt{\frac{12}{81}} = \sqrt{\frac{3\cdot 4}{3\cdot 27}} = \sqrt{\frac{4}{27}} =$ A student raises her hands and asks, "When we're doing this kind of problem, will it always be possible to cross multiply?"

07	Temperature Conversion	A	Setting: High school first-year Algebra class Task: Students were given the task of coming up with a formula that would convert Celsius temperatures to Fahrenheit temperatures, given that in Celsius 0° is the temperature at which water freezes and 100° is the temperature at which water boils, and given that in Fahrenheit 32° is the temperature at which water freezes and 212° is the temperature at which water boils. The rationale for the task is that if one encounters a relatively unfamiliar Celsius temperature, one could use this formula to convert to an equivalent, perhaps more familiar in the United States, Fahrenheit temperature (or vice versa). Mathematical activity that occurred: One student developed a formula based on reasoning about the known values from the two temperature scales. "Since 0 and 100 are the two values I know on the Celsius scale and 32 and 212 are the ones I know on the Fahrenheit scale, I can plot the points (0, 100) and (32, 212). If I have two points I can find the equation of the line passing through those two points. (0, 100) means that the y-intercept is 100. The change in y is (212-100) over the change in x, (32-0), so the slope is $\frac{112}{32}$. Since $\frac{112}{32} = \frac{7*16}{2*16}$, if I cancel the 16s the slope is $\frac{7}{2}$. So the formula is $y = \frac{7}{2}x + 100$."

08	Ladder Problem	G	A high school geometry class was in the middle of a series of lessons on loci. The teacher chose to discuss one of the homework problems from the previous day's assignment. A student read the problem from the textbook (Brown, Jurgensen, & Jurgensen, 2000): *A ladder leans against a house. As A moves up or down on the wall, B moves along the ground. What path is followed by midpoint M? (Hint: Experiment with a meter stick, a wall, and the floor.)* The teacher and two students conducted the experiment in front of the class, starting with a vertical "wall" and a horizontal "floor" and then marking several locations of M as the students moved the meter stick. The teacher connected the points. Their work produced the following data picture on the board:  A student commented, "That's a heck of an arc." Is it really an arc?

09	Perfect Square Trinomials	A	A teacher is teaching about factoring perfect square trinomials and has just gone over a number of examples. Students have developed the impression that they need only check that the first and last terms of a trinomial are perfect squares in order to decide how to factor it. They are developing the impression that the middle term is irrelevant. The teacher needs to construct a counterexample on the spot, and he wants one whose terms had no common factor besides 1.
10	Simultaneous Equations	A	A student teacher in a course titled *Advanced Algebra/Trigonometry* presented several examples of solving systems of three equations in three unknowns algebraically using the method of elimination (linear combinations). She started another example and had written the following $3x+5y-6z=-3$ $5x+y-2z=5$ when a student asked, “What if you only have two equations?”
11	Faces of a solid	G	Observing a 7th grade class, where the lesson was on classification of solids, the teacher held up a rectangular prism and asked the class how many "sides" there were. Two students responded, one with an answer of 12, the other with an answer of 6. The student with the answer of 6 was told that they were right and the lesson moved on. After the lesson was over, there was an opportunity to speaking with the student that gave the answer of 12. It was asked were he had gotten his answer. He was considering the edges, "sides". It was understandable that the student had made that misconception because the edges of polygons are also many times referred to as the sides. By not using the more correct term of faces, the teacher confused at least one student because of mathematical language.

12	Quadratic Equations	A	This situation occurred in the classroom of a student teacher during his student teaching. He worked very hard to create meaningful lessons for his students and often asked his mentor teacher for advice by asking questions similar to the one's found at the end of this vignette. Mr. Sing presents equations of the following to his students. $$(x+1)^2 = 9$$ He demonstrates to them that they need only take the square root of each side to get $x+1 = 3$ or $x+1 = -3$. Then we can solve for $x = 2$ or $x = -4$. He then turns his students loose to solve some equations like the ones he has presented and is surprised to find out that many of his students are multiplying the terms out to get $$x^2 + 2x + 1 = 9$$ and then transforming the equation so that $$x^2 + 2x - 8 = 0$$ and factoring this equation. Mr. Sing notes, however, that many students were making mistakes in carrying out his procedure. He stops the class and reminds the students that they need only take the square root of both sides to solve these types of equations and then let's them continue working on the problems. A few days later, Mr. Sing grades the test covering this material and finds that many of his students are still not doing as he has suggested. At first he thinks that his students just didn't

			listen to him but then he reminds himself that during the class period the students seemed to be quite attentive. What hypotheses do you have for why his students are acting in this way? What concepts might be necessary for students to understand the concept of solving a quadratic equation? In what ways might Mr. Sing work with his students to develop these concepts?
13	Trigonometric Equations	T	A student teacher was explaining how to solve trig equations of the form $\sin\theta = 0.06$ A debate occurred about what to do with 0.6. The student teacher said something like: "Take sine to the minus one on both sides of the equation". Then one student wanted to know whether this is like dividing by sine on both sides. *What responses might the student teacher consider?*

14	Factoring	A	Carrie was reviewing homework on factoring. One problem was $$x^3 - 5x^2 + x + 5 = (x+5)(-x^2+1)$$ Carrie factored the problem: $$(x+5)(-1)(x^2-1) = -(x+5)(x+1)(x-1)$$ The mentor teacher said, "Carrie, what are you doing? You need to rewrite $$(-x^2+1) = (1-x^2)$$ and factor." So the problem becomes $$(x+5)(1-x^2) = (x+5)(1+x)(1-x)$$ A student said that she did not understand why you could rewrite $$(-x^2+1)$$ as $$(1-x^2)$$ She said she never did that and did not know you could. *What might the student teacher and the mentor teacher do to clear up the confusion?*
15	Graphing Quadratic Functions	A	When preparing a lesson on graphing quadratic functions, a student teacher had many questions about teaching the lesson to a Concepts of Algebra class, an introductory algebra mathematics course. One of the concerns that the student teacher had was the graphing of the vertex of the parabola, which also means identifying the equation of the axis of symmetry. The textbook for this class claimed that $x = \frac{-b}{2a}$ was the equation of the line of symmetry. The student teacher wanted to know how to derive this equation.

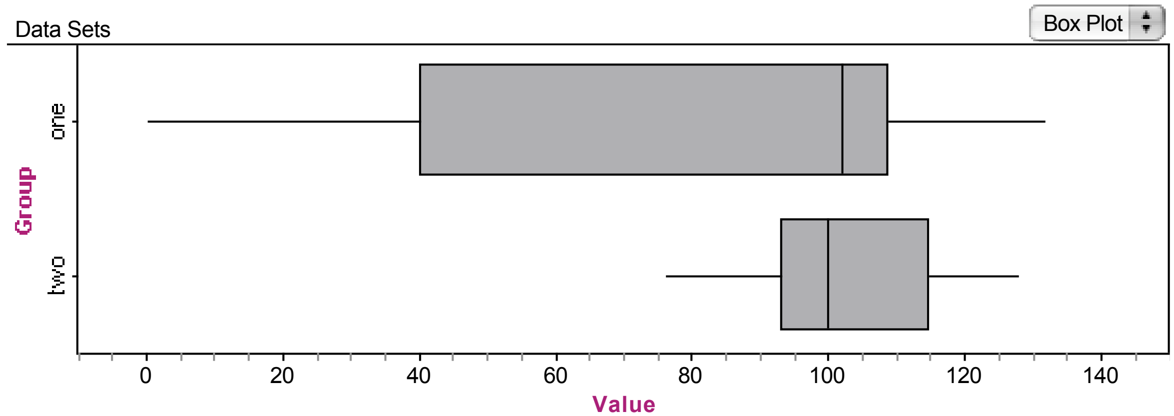
16	Area of Plane Figures	G	A teacher in a college preparatory geometry class defines the following formulas for areas of plane figures—the area of a triangle, square, rectangle, parallelogram, trapezoid, and rhombus. She removes the formulas from the overhead and poses several problems to the class of students, having students volunteer when they have the answer. One student seems to be particularly good at getting the answers correct but numerous other students struggle. Finally, a disgruntled student asks audibly to the whole class, "Man how did you [the student getting the correct answers] memorize those formulas so fast?" The other student responds, "I didn't memorize the formulas. I can just see what the area should be." *What ideas or focal points might the teacher use to capitalize on this interchange between students?*
17	Equivalent Equations	A	Students in a second year algebra class have been working on using graphs as one tool in solving quadratic equations. When the students were solving linear equations, the teacher placed a lot of emphasis on generating and recognizing equivalent equations (e.g., $2x + 6 = 18$ is equivalent to $x = 6$), but the students did not graph these equations to solve them. In their current work, one group of students contend that $2x^2 - 6x = 20$ cannot be equivalent to $x^2 - 3x - 10 = 0$ because the graphs don't look the same—in fact in graphing the first equation, you have to graph $y = 2x^2 - 6x$ and the line $y = 20$, while in the second you graph $y = x^2 - 3x - 10$ and the line $y = 0$ (which you don't really have to graph since it's just the x-axis). *What kind of mathematical knowledge does the teacher need to consider in responding to these students?*
18	Exponential Rules	A	Students in an algebra class have just finished a unit on exponential powers, including standard exponential rules and negative exponents. In completing a sheet of true/false questions, most of the students have classified the following statement as false: $2^{17} + 2^{17} = 2^{18}$. *What kind of mathematical knowledge does the teacher need to consider in responding to the work of the students?*

20	Exponential Rules	A	In an Algebra II class, students had just finished reviewing the rules for exponents. The teacher wrote $x^m \bullet x^n = x^5$ on the board and asked the students to make a list of values for *m* and *n* that made the statement true. After a few minutes, one student asked, "Can we write them all down? I keep thinking of more."
21	Exponential Rules	A	In an Algebra II class, students had just finished reviewing the rules for exponents. The teacher wrote on the board and asked the students to make a list of values for *m* and *n* that made the statement true. After a few minutes, one student asked, "Can we write them all down? I keep thinking of more."
22	Operations with Matrices	A	Students in an Algebra II class had been discussing the addition of matrices and had worked on several examples of n x n matrices. Most were proficient in finding the sum of two matrices. Toward the end of the class period, the teacher announced that they were going to being working on the multiplication of matrices, and challenged the students to find the product of two 3 x 3 matrices: $$\begin{bmatrix} 2 & 4 & 5 \\ 5 & 2 & 3 \\ 1 & 4 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 5 \\ 5 & 2 & 3 \end{bmatrix}$$ Students began to work on the problem by multiplying each corresponding term in a way similar to how they had added terms. One student shared his work on the board getting a product of $$\begin{bmatrix} 2 & 12 & 10 \\ 10 & 12 & 15 \\ 5 & 8 & 12 \end{bmatrix}$$ As the period ended, the teacher asked students to return to the next period with comments about the proposed method of multiplying and alternative proposals.

23	Simultaneous Equations	A	A mentor teacher and student teacher are discussing a student teacher's lesson after it has been taught and the mentor is encouraging the student teacher to probe student thinking and to ask good questions. The class was solving simultaneous equations and the student teacher had chosen the following pair of equations to discuss: $y = 2/3\,x + 4$ and $42 = 4x - 6y$ A high school student had responded that there was not a common solution because they were parallel, and the student teacher had moved to the next problem. The mentor praised the student teacher for picking this pair which had no common solution, but urged him to ask follow-up questions. **Question:** What questions could a teacher ask that would help students understand more about solutions to simultaneous equations and what it means not to have a solution? (What would be a good set of pairs of equations for a class to study and why?)
24	Isn't Absolute Value Always Positive	A	In a first-year algebra course, a discussion centered on solving absolute value inequalities. One student said that since everything in $\|x + 3\| < 5$ is positive, the solution could be found by solving $x + 3 < 5$, so the solution was $x < 2$.

26	Absolute Value	A	A student teacher begins a tenth-grade geometry lesson on solving absolute value equations by reviewing the meaning of absolute value with the class. They discussed that the absolute value represents a distance from zero on the number line and that the distance cannot be negative. He then asks the class what the absolute value tells you about the equation $\|x\|=2$. To which a male student responds "anything coming out of it must be 2". The student teacher states "*x* is the distance of 2 from 0 on the number line". Then on the board, the student teacher writes $\|x+2\|=4$ $x+2=4 \quad and \quad x+2=-4$ $x=2 \qquad\qquad x=-6$ And graphs the solution on a number line. A puzzled female student asks, "Why is it 4 and –4? How can you have –6? You said that you couldn't have a negative distance?"
27	Product of Two Negative Numbers	N	A question commonly asked by students in middle school and secondary mathematics classes is "Why is it that when you multiply two negative numbers together you get a positive number answer?"
28	Adding Radicals	N	Mr. Fernandez is bothered by his ninth-grade algebra students' responses to a recent quiz on radicals, specifically those in response to a question about square roots in which students added $\sqrt{2}$ and $\sqrt{3}$ and got $\sqrt{5}$.

29	Trigonometric Identities	T	While proving trigonometric identities such as $\sin x \cdot \cos x \cdot \tan x = \frac{1}{\csc^2 x}$, a student's work is this: $\sin x \cdot \cos x \cdot \tan x = \frac{1}{\csc^2 x}$ $\csc^2 x \cdot \sin x \cdot \cos x \cdot \tan x = 1$ $\frac{1}{\sin^2 x} \cdot \sin x \cdot \cos x \cdot \frac{\sin x}{\cos x} = 1$ $\cos x \cdot \frac{1}{\cos x} = 1$ $1 = 1$ *Is this a proper proof of the trigonometric identity? If not, how would you explain to the student the mistake with the proof?*
30	Translations of Functions	A	During a unit on functions, the transformation of functions from their parent function is discussed in a class. For example, if the parent function is $y = x^2$, then the child function $y = x^2 + 4$ would have a vertical translation of 4 units. When the class encounters the function $y = (x-2)^2 + 3$, one student notes that the vertical translation of +3 "makes sense," but the horizontal translation to the right of 2 does not "make sense" with a – 2 within the function. As a teacher, how would you explain this?
31	Expanding Binomials	A	In a high school Algebra I class, students were given the task of expanding $(x + 5)^2$. A student responds, "That's easy! Doesn't $(x+5)^2 = x^2 + 25$?"

33	Least Squares Regression	S	During a discussion of lines of best fit, a student asks why the sum of the squared differences between predicted and actual values is used. Why use squared differences to find the line of best fit? Why use differences rather than some other measure to find the line of best fit?
34	Mean Median	S	The following task was given to students at the end of the year in an AP Statistics class. Consider the following box plots and five-number summaries for two distributions. Which of the distributions has the greater mean? 

Data Sets

	Group	
	one	two
Value	0	76
	40	93
	102	100
	109	115
	132	128

S1 = min ( )
S2 = Q1 ( )
S3 = median ( )
S4 = Q3 ( )
S5 = max ( )

One student's approach to this problem was to construct the following probability distributions for each data set, and then to compare the corresponding expected values to determine which data set has the greater mean. The student responded that Data set two had the larger mean.

Data set one:

E(X) = 79.25

	0 - 40	40 - 102	102 - 109	109 - 132
X	20	71	105.5	120.5
P(X)	0.25	0.25	0.25	0.25

Data set two:

E(X) = 102.75

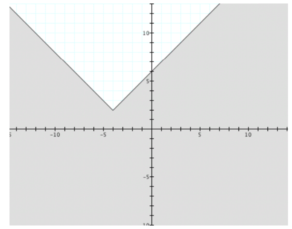
	76-93	93-100	100-115	115-128
X	84.5	96.5	107.5	121.5
P(X)	0.25	0.25	0.25	0.25

35	Quadratic Equation	A	In an Algebra 1 class some students began solving a quadratic equation as follows: Solve for x: $x^2 = x + 6$ $\sqrt{x^2} = \sqrt{x+6}$ $x = \sqrt{x+6}$ They stopped at this point, not knowing what to do next.
36	Pythagorean Theorem	G	The mathematical paths described occurred in a high school Algebra I course and again in an Advanced Algebra course that I taught. The goal for the lesson was to discover the Pythagorean theorem. Students were given transparency cutouts of graph paper squares with side lengths from one unit to twenty-five units. Students were asked to create triangles whose sides had the side-lengths of three squares. Students worked through the activity and with some prompting began to notice the squares that would create right triangles and the relationship involving the area of those squares. A student asked, “Does this work for every right triangle?”

37	Distributing Exponents	A	The following scenario took place in a high school Algebra 1 class. Most of the students were sophomores or juniors repeating the course. During the spring semester, the teacher had them do the following two problems for a warm-up: 1) Are the two expressions, $\left(x^3y^5\right)^2$ and x^6y^{10}, equivalent? Why or why not? 2) Are the two expressions, $(a+b)^2$ and a^2+b^2, equivalent? Why or why not? Roughly a third of the class stated that both pairs of expressions were equivalent because of the Distributive Property.
38	Irrational Length	G	A secondary pre-service teacher was given the following task to do during an interview: Given: square ABCD. Construct a square whose area is half the area of square ABCD. (Note: The pre-service teacher was not given a drawing or any dimensions for ABCD.) The student chose the dimensions of ABCD to be 1 unit by 1 unit and approached the problem in two ways.

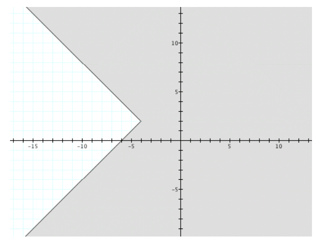
39	Summing the Natural Numbers	N	The course was a mathematical modeling course for prospective secondary mathematics teachers. The discussion focused on finding an explicit formula for a sequence expressed recursively. During the discussion, the students expressed a need to sum the natural numbers from 1 to n. After several attempts to remember the formula, a student hypothesized that the formula contained n and $n + 1$. Another student said he thought it was $\frac{n(n+1)}{2}$ but was not sure. During the ensuing discussion, a third student asked, “How do we know that $\frac{n(n+1)}{2}$ is the sum of the integers from 1 to n? And won’t that formula sometimes give a fraction?”
40	Powers	A	During an Algebra I lesson on exponents, the teacher asked the students to calculate positive integer powers of 2. A student asked the teacher, “We’ve found 2^2 and 2^3. What about $2^{2.5}$?”
41	Square Roots	A	A teacher asked her students to sketch the graph of $f(x)=\sqrt{-x}$. A student responded, “That’s impossible! You can’t take the square root of a negative number!”
42	Sin (2x)	T	During a lesson on transformations of the sine function a student asks, “Why is the graph of $y = \sin 2x$ a horizontal shrink of the graph of $y = \sin x$ instead of a horizontal stretch?”
43	Can You Circumscribe a Circle about This Polygon?	G	In a geometry class, after a discussion about circumscribing circles about triangles, a student asked, “Can you circumscribe a circle about any polygon?”

44	Zero Exponents	A	In an Algebra I class, a student questions the claim that $a_0 = 1$ for all non-zero real number values of a. The student asks, "How can that be possible? I know that a_0 is a times itself zero times, so a_0 must be zero."
45	Zero-Product Property	A	A student in Algebra I class wrote the following solution on a homework problem: x^2 - 4x - 5 = 7 (x -5)(x + 1) = 7 x - 5 = 7 x + 1 = 7 x = 12 x = 6 A different student commented that 6 indeed was solution to the equation since 6^2 - 4(6) – 5 =7, but that 12 was not.
46	Division Involving Zero	N	On the first day of class, preservice middle school teachers were asked to evaluate $\frac{2}{0}, \frac{0}{0}, \ and \ \frac{0}{2}$ and to explain their answers. There was some disagreement among their answers for $\frac{0}{0}$ (potentially 0, 1, undefined, and impossible) and quite a bit of disagreement among their explanations: • Because any number over 0 is undefined; • Because you cannot divide by 0; • Because 0 cannot be in the denominator; • Because 0 divided by anything is 0; and • Because a number divided by itself is 1.
47	Graphing Inequalities with Absolute Values	A	This episode occurred during a course for prospective secondary mathematics teachers. The discussion focused on the graph of $y - 2 \leq \lvert x + 4 \rvert$. The instructor demonstrated how to graph this inequality using compositions of transformations, generating the following graph.

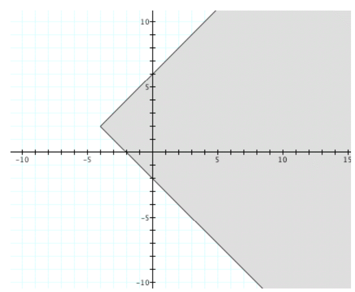


Students proposed other methods, which included the two different algebraic formulations and accompanying graphs as seen below.

$$y-2 \le x+4 \text{ or } -x-4 \le y-2$$



$$y-2 \le x+4 \text{ and } -x-4 \le y-2$$



Students expected their graphs to match the instructor's graph, and they were confused by the differences they saw.

48	The Product Rule for Differentiation	C	In an introductory calculus classroom, a student asks the teacher the following question: "Why isn't the derivative of $y = x^2 \sin x$ just $y' = 2x \cos x$?
49	Similarity	G	In a geometry class, students were given the diagram in Figure 1 depicting two acute triangles, ΔABC and $\Delta A'B'C'$, and students were told that $\Delta ABC \sim \Delta A'B'C'$ with a figure (Figure 1) indicating that $A'B' = 2AB$ and $m\angle B = 75°$. From this, a student concluded that $m\angle B' = 150°$.
50	Connecting Factoring with the Quadratic Formula	A	Mr. Jones suspected his students saw no direct connection between the work they had done on factoring quadratics and the quadratic formula.
51	Proof by Induction	A	A teacher of a calculus course gave her students the opportunity to earn extra credit by proving various algebraic formulas by mathematical induction. For example, one of the formulas was the following: $1^2 + 2^2 + \ldots + n^2 = (n)(n + 1)(2n + 1)/6$. Only one student in the class was able to prove any of the formulas. After the student had presented his three proofs to the class on three consecutive days, another student complained: "I don't get what he is proving. And besides that, how do you get the algebraic formulas to start with?"
55	Multiplication of Complex Numbers	A	The teacher of an Algebra III course notices that her students are having difficulty understanding some of the differences between the multiplication of real numbers, on the one hand, and the multiplication of complex numbers, on the other. The teacher wants her students to have a feasible way of thinking about the multiplication of complex numbers. In particular, she wants them to see these new numbers as having properties that are different from those of the real numbers.

62	Absolute Value and Square Roots	A	A talented 7th-grade student was working on the task of producing a function that had certain given characteristics. One of those characteristics was that the function should be undefined for values less than 5. Another characteristic was that the range of the function should contain only non-negative values. In the process, he defined $f(x) = \left\|\sqrt{x-5}\right\|$ and then evaluated $f(-10)$. The result was 3.872983346. He looked at the calculator screen and whispered, "How can that be?"
65	Square Root of *i*	A	Knowing that a Computer Algebra System (CAS) had commands such as **cfactor** and **csolve** to factor and solve complex numbers respectively, a teacher was curious about what would happen if she entered $\sqrt{i}$. The result was $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$. Why would a CAS give a result like this?